COMP50001: Algorithm Design & Analysis

Sheet 1 (Week 2)

Exercise 1.1

Given the following function concatenating two lists,

$$
\begin{aligned}
&(+\!\!+) :: [Int] \to [Int] \to [Int] \\
&[\,] \quad\quad\;\; +\!\!+\; ys = ys \\
&(x : xs) +\!\!+\; ys = x : (xs +\!\!+\; ys)
\end{aligned}
$$

with a recurrence relation $T(n, m)$, approximate the time it takes to compute $xs +\!\!+\; ys$ for any list xs of length n and ys of length m.

Exercise 1.2

Consider an alternative strict time analysis function T', defined to be the same as T, except that T' is refined to have cost 1 instead 0 on variables, constants and primitive functions, i.e.

$$
\begin{aligned}
T'(x) &= 1 \\
T'(k) &= 1 \\
T'(f)\ x_1\ \cdots x_n &= 1
\end{aligned}
$$

Compute $T'(length\ xs)$ in terms of $T'(length\ (tail\ xs))$.

Exercise 1.3

Compute the strict running time $T(length\ \ (insert\ \ x\ \ xs))$ using the composition rule.

Exercise 1.4

Pattern matching can be added to the expression language e as follows:

$$e ::= \cdots \mid \mathbf{case}\ e\ \mathbf{of}\ [\,] \to e; (x : xs) \to e$$

Give an appropriate definition of $T(\mathbf{case}\ e_1\ \mathbf{of}\ [\,] \to e_2; (x : xs) \to e_3)$ for strict time analysis.

Exercise 1.5

(ADWH, p39, Exercise 2.3) Prove formally that $(n+1)^2 \in \Theta(n^2)$ by exhibiting the necessary constants.

Exercise 1.6

(ADWH, p39, Exercise 2.5) Justify whether each of the following is true or false:

1. $2n^2 + 3n \in \Theta(n^2)$
2. $2n^2 + 3n \in O(n^3)$

3. $n \log n \in O(n\sqrt{n})$
4. $n + \sqrt{n} \in O(\sqrt{n}\log n)$
5. $2^{\log n} \in O(n)$

Exercise 1.7

Show formally that $o(g(n))$ is a proper subset of $O(g(n))$ for any function g using their definitions.

Exercise 1.8

Explain why there is no definition $\theta(g(n))$ that corresponds to $\Theta(g(n))$ even though there is $o(g(n))$ corresponding to $O(g(n))$ and $\omega(g(n))$ corresponding to $\Omega(g(n))$.